

Pomeranchuk-Nematic instability in the presence of a weak magnetic field

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We analyze a two-dimensional Pomeranchuk-Nematic instability, trigger by the Landau parameter $F_2 < 0$, in the presence of a small magnetic field. Using Landau Fermi liquid theory in the isotropic phase, we study the collective modes near the quantum critical point $F_2 = -1, \omega_c = 0$ (where ω_c is the cyclotron frequency). We focus on the effects of parity symmetry breaking on the Fermi surface deformation. We show that, for studding the critical regime, the linear response approximation of the Landau-Silin equation is not sufficient and it is necessary to compute corrections at least of order ω_c^2 . Identifying the slowest oscillation mode in the disordered phase, we compute the phase diagram for the isotropic/nematic phase transition in terms of F_2 and ω_c .

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I. INTRODUCTION

The isotropic-nematic quantum phase transition was proposed as a possible mechanism to explain the anisotropic behavior of several strongly correlated systems. Some interesting examples are quantum Hall liquids, high T_c superconductors and heavy fermions systems. An interesting review can be found in reference [1].

This transition can be understood as an instability of a Fermi surface under the influence of a strongly attractive two-body potential in the forward scattering channel, with d-wave symmetry (or equivalently, with angular momentum $\ell = 2$). From the point of view of Landau Fermi liquid theory, it is triggered by a Pomeranchuk instability produced by a large negative value of the Landau parameter F_2 in the charged sector. As a consequence of the transition, the Fermi surface is deformed, getting and ellipsoidal component. The Goldstone modes, related with rotational symmetry breaking, are dissipative over-damped excitations, characterized by dynamical exponent $z = 3$. The order parameter theory was developed using different techniques: mean field theory², multidimensional bosonization^{3,4} and Landau Fermi liquid theory⁵. While the collective bosonic excitations are reasonable well understood, the fate of the fermionic spectrum is still under debate^{4,6,7}.

From an experimental point of view, the study of Fermi surface deformations can be performed by means of at least two independent techniques: ARPES⁸ and the observation of quantum oscillations⁹, like for instance, the de Haas-van Alphen effect. The use of the latter resides in the ability to reconstruct Fermi surface shapes from the information contained in quantum oscillations of different observables, when an externally applied magnetic field is varied.

The application of a strong magnetic field suppress any Pomeranchuk instability, since it opens a gap in the spectrum due to Landau level quantization. However, for small magnetic fields, the Landau levels form a dense set near the Fermi energy and strong attractive interactions

mix all levels in a non-trivial way. Therefore, it is important to understand the critical behavior when the quantum critical point is reached by lowering the magnetic field.

With this motivation, we would like to present a study of a two-dimensional Pomeranchuk-nematic instability in the presence of a small magnetic field, applied perpendicular to the two-dimensional fermionic system. We have considered an isotropic and homogeneous charged Fermi liquid submitted to a small magnetic field, $k_B T \ll \hbar \omega_c \ll \epsilon_F$, where $\omega_c = eB/m^*$ is the cyclotron frequency and ϵ_F is the Fermi energy of the system. We have focused on a simplified model where only the attractive two-body d-wave interaction is present. Using a semi-classical approach, we have studied collective excitations of the fermionic system using the Landau-Silin equation^{10,11}. Studding the oscillatory slowest mode, we can compute the transition line where the isotropic phase gets unstable. The main result is presented in figure (1) where we depict the phase diagram from the nematic-Pomeranchuk instability. In this figure the horizontal axis is the usual Landau control parameter $\alpha = 1 + F_2$ while the vertical axis is the adimensional magnetic field $(\omega_c/v_F q)^2$. The $v_F q$ term is the energy of a typical large distance disturbance of the Fermi surface with momentum $q \ll p_F$. We observe a maximum value of the magnetic field above which no Pomeranchuk instability is possible. Moreover, we have observed a reentrant behavior of the isotropic phase for greater values of the interaction parameter. We have also analyzed the behavior of collective modes near the quantum critical point ($F_2 \rightarrow -1, \omega_c \rightarrow 0$). Since the magnetic field breaks parity symmetry, the collective mode dynamics mixes symmetric as well as antisymmetric modes. Then, the Fermi surface deformation is not an ellipsoid but has a definite parity given by the direction of the magnetic field and the momentum \mathbf{q} of the periodic perturbation.

The paper is structured as follows: in section II we briefly review the Landau theory of charged Fermi liquids and the Landau-Silin equation to describe the collective modes of a Fermi liquid submitted to an external

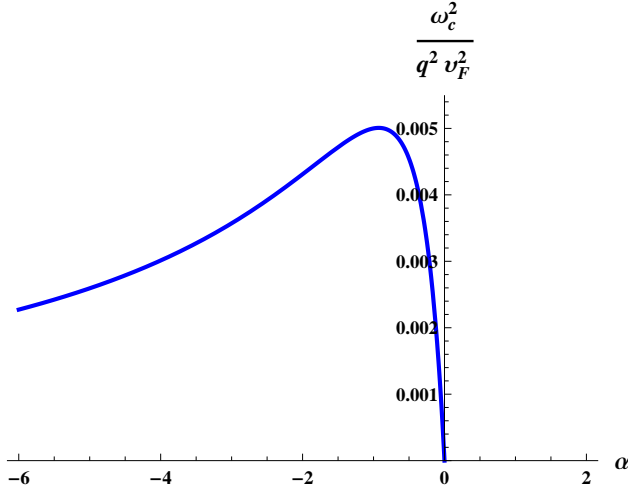


FIG. 1. Phase diagram for the Pomeranchuk instability $\ell = 2$ in the presence of a small magnetic field. The external part of the transition curve represents an isotropic Fermi liquid, while the inner part is an anisotropic phase. The control parameters are $\alpha = 1 + F_2$ and $\omega_c = \frac{eB}{m^*}$. We observe, under a maximum value of the magnetic field, a re-entrant behavior as far as the control parameter α is varied.

magnetic field. In §III we set our model and deduce the phase diagram of figure (1). In §IV we show the collective modes near the nematic quantum critical point. Finally, we discuss our results and we point out possible future developments in §V.

II. SEMI-CLASSICAL APPROXIMATION

Following the standard Fermi liquid approach¹², we start by writing down the energy functional for a two-dimensional system of spin-less quasi-particles of effective mass m^* , in an electromagnetic field defined by the vector potential $\mathbf{A} = \mathbf{A}(\mathbf{r}, t)$,

$$E[n] = \sum_{\mathbf{p}} \frac{(\mathbf{p} + e\mathbf{A})^2}{2m^*} n(\mathbf{p}, \mathbf{r}) + \sum_{\mathbf{p}, \mathbf{p}'} \int d\mathbf{r} d\mathbf{r}' f_{\mathbf{p}+e\mathbf{A}; \mathbf{p}'+e\mathbf{A}}(\mathbf{r} - \mathbf{r}') \times n(\mathbf{p}, \mathbf{r}) n(\mathbf{p}', \mathbf{r}) + \mathcal{O}(n^3), \quad (1)$$

where $n(\mathbf{p}, \mathbf{r})$ is the phase-space density at momentum \mathbf{p} and position \mathbf{r} . e is the quasi-particle charge and $f_{\mathbf{p}+e\mathbf{A}; \mathbf{p}'+e\mathbf{A}}(\mathbf{r} - \mathbf{r}')$ is the Landau amplitude characterizing finite range two-particle interactions. The Landau interaction function should depend on the electromagnetic vector potential to guarantee gauge invariance.

In order to compute a semi-classical evolution equation, we define the effective single-particle Hamiltonian

$$H_{\text{eff}}(\mathbf{p}, \mathbf{r}) = \frac{\delta E[n]}{\delta n(\mathbf{p}, \mathbf{r})}, \quad (2)$$

which generates the following time evolution equation:

$$\frac{\partial n(\mathbf{p}, \mathbf{r}, t)}{\partial t} = \{H_{\text{eff}}, n(\mathbf{p}, \mathbf{r}, t)\}_{\text{PB}} + I_{\text{coll}}[n(\mathbf{p}, \mathbf{r}, t)], \quad (3)$$

where $\{\dots\}_{\text{PB}}$ are Poisson brackets associated with the conjugate variables \mathbf{r} and \mathbf{p} and the effects of quasi-particle scattering are included in the collision integral $I_{\text{coll}}[n(\mathbf{p}, \mathbf{r}, t)]$.

By means of the Hamilton's equations of motion $d\mathbf{r}/dt = \nabla_{\mathbf{p}} H_{\text{eff}}(\mathbf{p}, \mathbf{r}, t)$ and $d\mathbf{p}/dt = -\nabla_{\mathbf{r}} H_{\text{eff}}(\mathbf{p}, \mathbf{r}, t)$, it is obtained the so-called Landau-Silin kinetic equation¹¹

$$\begin{aligned} \frac{\partial n(\mathbf{p}, \mathbf{r}, t)}{\partial t} + \mathbf{v}(\mathbf{p}, \mathbf{r}, t) \cdot \nabla_{\mathbf{r}} n(\mathbf{p}, \mathbf{r}, t) - (\mathcal{F}(\mathbf{p}, \mathbf{r}, t) \\ + \sum_{\mathbf{p}'} \int d\mathbf{r}' f_{\mathbf{p}+e\mathbf{A}; \mathbf{p}'+e\mathbf{A}}(\mathbf{r} - \mathbf{r}') \nabla_{\mathbf{r}'} n(\mathbf{p}', \mathbf{r}', t)) \cdot \\ \cdot \nabla_{\mathbf{p}} n(\mathbf{p}, \mathbf{r}, t) = I_{\text{coll}}[n(\mathbf{p}, \mathbf{r}, t)], \end{aligned} \quad (4)$$

where $\mathcal{F}(\mathbf{p}, \mathbf{r}, t) = e[\mathbf{E}(\mathbf{r}, t) + \mathbf{v}(\mathbf{p}, \mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t)]$ is the Lorentz force and $\mathbf{v}(\mathbf{p}, \mathbf{r}, t) = \nabla_{\mathbf{p}} H_{\text{eff}}$ is the quasi-particle velocity, including interactions. The Landau-Silin transport equation (4) resembles the conventional classical Boltzmann equation. However, the effective Lorentz force $\mathcal{F}(\mathbf{k}, \mathbf{r}, t)$ depends self-consistently on the quasi-particles distribution function $n(\mathbf{p}, \mathbf{r}, t)$.

In this paper we want to study the effect of an external magnetic field B , applied perpendicular to the plane of the system. We will assume that the cyclotron energy $\hbar\omega_c = \hbar eB/m^* \ll \varepsilon_F$. For simplicity through the paper we choose $\hbar \equiv 1$. In general, the scattering mechanisms described by the collision integral can be studied applying the relaxation-time τ approximation. We will consider that the typical collective mode frequencies are greater than the collision quasi-particle frequency. Of course, this is not true at criticality. However, to determine the position of the transition line, it is enough to consider $I_{\text{coll}} \rightarrow 0$. To set up the kinetic equation of the Fermi surface collective modes, let us consider a constant isotropic equilibrium distribution n_p^0 and a small perturbation δn , such that $n(\mathbf{p}, \mathbf{r}, t) = n_p^0 + \delta n(\mathbf{p}, \mathbf{r}, t)$. In these conditions, the linear expansion of equation (4) in δn provides the transport equation

$$\frac{\partial \delta n}{\partial t} + \mathbf{v}_{\mathbf{p}}^0 \cdot \nabla_{\mathbf{r}} \delta \bar{n} - e[\mathbf{v}_{\mathbf{p}}^0 \times \mathbf{B}] \cdot \nabla_{\mathbf{p}} \delta \bar{n} = 0, \quad (5)$$

where

$$\begin{aligned} \delta \bar{n}(\mathbf{p}, \mathbf{r}, t) = \delta n(\mathbf{p}, \mathbf{r}, t) - \left(\frac{\partial n^0}{\partial \varepsilon^0} \right) \times \\ \times \sum_{\mathbf{p}'} \int d\mathbf{r}' f_{\mathbf{p}+e\mathbf{A}; \mathbf{p}'+e\mathbf{A}}(\mathbf{r} - \mathbf{r}') \delta n(\mathbf{p}', \mathbf{r}', t) \end{aligned} \quad (6)$$

is the deviation from local equilibrium.

It is important to point out that equation (5) is linear in δn_p however, is highly non-linear in the magnetic field since it enters the definition of the Landau interaction

(equation (6)). Usually, to compute collective plasma modes in charged Fermi liquids this last contribution is neglected ended with a true linear response theory¹³. However, as we will show, this approximation is not consistent to study Pomeranchuk instabilities.

At low temperatures $k_B T \ll \epsilon_F$ the electron dynamics is confined to a small region around the Fermi surface. Then, it is more convenient to define $\delta n(\mathbf{p}, \mathbf{r}, t) = -(\partial n_{\mathbf{p}}^0 / \partial \epsilon_{\mathbf{p}}) \nu_{\mathbf{p}}(\mathbf{r}, t)$, where $\nu_{\mathbf{p}}(\mathbf{r}, t)$ measures local Fermi surface deformation. Finally, Fourier transforming in the space variable \mathbf{r} , the kinetic equation (5) becomes,

$$\frac{\partial \nu_{\mathbf{p}}(\mathbf{q}, t)}{\partial t} + (i \mathbf{v}_F^0 \cdot \mathbf{q} - e(\mathbf{v}_F^0 \times \mathbf{B}) \cdot \nabla_{\mathbf{p}}) \times (\nu_{\mathbf{p}}(\mathbf{q}, t) + \delta \epsilon_{\mathbf{p}}(\mathbf{q}, t)) = 0, \quad (7)$$

where \mathbf{v}_F^0 is the Fermi velocity and the expression

$$\delta \epsilon_{\mathbf{p}}(\mathbf{q}, t) = \frac{1}{V^2} \sum_{\mathbf{p}'} \left(\frac{\partial n_{\mathbf{p}'}^0}{\partial \epsilon_{\mathbf{p}'}} \right) \int d\mathbf{r} d\mathbf{r}' e^{i\mathbf{q} \cdot \mathbf{r}} \times \times f_{\mathbf{p}+e\mathbf{A}, \mathbf{p}'+e\mathbf{A}}(\mathbf{r} - \mathbf{r}') \nu_{\mathbf{p}'}(\mathbf{r}', t) \quad (8)$$

describes the combined effect of interactions and magnetic field, being V the space volume.

Equations (7) and (8) are the starting point of our analysis. In the next section we will set up our model and will study the Pomeranchuk instability in the nematic channel.

III. THE POMERANCHUK NEMATIC INSTABILITY

For simplicity we consider a two-dimensional circular Fermi surface. The interaction Landau function depends then essentially on the angle between two Fermi momenta and can be expanded in Landau parameters as

$$f_{\mathbf{p}+e\mathbf{A}, \mathbf{p}'+e\mathbf{A}}(\mathbf{r} - \mathbf{r}') \rightarrow f_{p_F, p_F'}(r) = \sum_{\ell} f_{\ell}(r) P_{\ell}(\cos \Phi), \quad (9)$$

where P_{ℓ} are Legendre polynomials and, in the absence of magnetic field, $\cos(\Phi) = \mathbf{p}_F \cdot \mathbf{p}'_F / p_F^2$. Moreover, we can expand the deformation of the Fermi surface in Fourier coefficients

$$\nu_{\mathbf{p}}(\mathbf{q}, t) = \sum_{\ell} \nu_{\ell}(q, t) e^{i\ell\theta} \quad (10)$$

where $\cos(\theta) = \mathbf{p}_F \cdot \mathbf{q} / p_F q$.

To study the Pomeranchuk-nematic instability, it is sufficient to consider a simplified model defined by $f_2(r) \neq 0$, while $f_{\ell}(r) = 0$ for all $\ell \neq 2$. The presence of other interaction channels does not modify our results qualitatively, provided they are all stable^{4,5}. We will consider a short-ranged but non-local interaction $f_2(r)$, whose Fourier transform of is given by

$$\tilde{f}_2(q) = \frac{f_2}{1 + |F_2|(\kappa q)^2}, \quad (11)$$

where $F_2 = N(0)f_2$ is the usual adimensional Landau parameter with angular momentum $\ell = 2$ ($N(0)$ is the density of states at the Fermi surface), and κ defines an effective interaction range $\xi = \sqrt{|F_2|}\kappa$. Our approach is valid provided $p_F^{-1} \ll \xi \ll q^{-1}$ i. e., when the interaction range is much larger than the inter-particle distance, however shorter than the typical scale of long-ranged perturbations.

In the absence of a magnetic field, the collective dynamics of the Fermi surface, given by equation (7) with $\mathbf{A} = 0$, reduces to

$$\frac{\partial \nu_{\ell}(q, t)}{\partial t} + \frac{i v_F q}{2} [\alpha_{\ell-1} \nu_{\ell-1}(q, t) + \alpha_{\ell+1} \nu_{\ell+1}(q, t)] = 0, \quad (12)$$

where we have defined $\alpha_{\ell} = 1 + F_{\ell}$ and F_{ℓ} are adimensional Landau parameters. In our model $\alpha_2 \equiv \alpha = 1 + F_2$ and $\alpha_{\ell} = 1$ for all $\ell \neq 2$.

We can gain more physical insight by defining symmetric and antisymmetric variables,

$$\nu_{\ell}^{\pm}(q, t) = \frac{1}{2} [\nu_{\ell}(q, t) \pm \nu_{-\ell}(q, t)] \quad (13)$$

in terms of which, the Fermi surface deformations are parametrized as

$$\nu(\mathbf{q}, \theta, t) = \sum_{\ell=0}^{\infty} \nu_{\ell}^{+}(q, t) \cos(\ell\theta) + \sum_{\ell=1}^{\infty} \nu_{\ell}^{-}(q, t) \sin(\ell\theta). \quad (14)$$

Eliminating in equation (12) odd components in favor of even ones, we obtain the coupled oscillator equations⁵

$$\frac{\partial^2 \nu_{\ell}^{\pm}(q, t)}{\partial t^2} + \left(\frac{v_F q}{2} \right)^2 [A_{\ell} \nu_{\ell}^{\pm}(q, t) + C_{\ell-1} \nu_{\ell-2}^{\pm}(q, t) + C_{\ell+1} \nu_{\ell+2}^{\pm}(q, t)] = 0 \quad (15)$$

with the adimensional coefficients,

$$A_{\ell} = \alpha_{\ell}(\alpha_{\ell-1} + \alpha_{\ell+1}) \\ C_{\ell} = \alpha_{\ell+1} \sqrt{\alpha_{\ell} \alpha_{\ell-1}}. \quad (16)$$

It is clear from equation (15) that the even and odd components of ℓ are decoupled. The same happens with the symmetric and antisymmetric components. The physical reason for that is parity invariance. Hence, the ν_2^{+} mode is coupled with the even symmetric modes $\nu_0, \nu_4^{+}, \nu_6^{+}, \dots$. Near $F_2 = -1$, or $\alpha \sim 0$, the ν_2^{+} mode oscillates with frequency

$$\omega_2 = \sqrt{2\alpha} \left(\frac{v_F q}{2} \right), \quad (17)$$

while all the other modes essentially oscillates with $\omega_{\ell} \sim v_F q / \sqrt{2}$. Then, near $\alpha = 0$, $\omega_2 \ll \omega_{\ell}$ with $\ell \neq 2$ showing that, in time scales $\tau \gg (v_F q)^{-1}$, ν_2^{+} is a very slow mode, while all other rapid modes can be averaged to zero. Therefore, when $\alpha \rightarrow 0$, the Fermi surface has an essentially elliptic form during long periods of time. This is the onset of the Pomeranchuk-Nematic instability.

When a magnetic field is applied parity, as well as, time reversal symmetry are broken. Then, the symmetric and antisymmetric modes are no longer decoupled. In linear response theory, we can ignore the contribution of the magnetic field in equation (8), then, equation (7) can be simplified to,

$$\frac{\partial \nu_\ell}{\partial t} + \frac{iv_F q}{2} [\alpha_{\ell-1} \nu_{\ell-1} + \alpha_{\ell+1} \nu_{\ell+1}] + i\alpha_\ell \omega_c \nu_\ell = 0, \quad (18)$$

where we have defined the cyclotron frequency $\omega_c = eB/m^*$. Thus, the linear response correction to equation (12) is proportional to $\alpha_\ell \omega_c$, where $\alpha_\ell = 1 + F_\ell$. Since $\alpha_\ell \sim 1$ for stable modes ($\ell \neq 2$), this equation is suitable to study collective modes of the Fermi liquid in small magnetic fields. However, near the Pomeranchuk instability ($\alpha_2 \equiv \alpha \sim 0$), ω_c^2 is of the same order of $\alpha \omega_c$ and cannot be ignored. To see this more clearly, we can compute the oscillation frequency of ν_2^+ , using equation (18), obtaining,

$$\omega_2 \sim \sqrt{2\alpha} \left(\frac{v_F q}{2} \right) \left\{ 1 + 4\alpha \left(\frac{\omega_c}{v_F q} \right)^2 + \dots \right\}, \quad (19)$$

where the ellipsis “...” mean terms of order $O(\alpha^2(\omega_c/v_F q)^4)$. Clearly, for small $\alpha \ll 1$, the frequency is approximately given by equation (17) without changing the behavior of the quantum critical point $\alpha = 0$.

Therefore, to study the transition line $\omega_c(\alpha)$, we need to consider quadratic corrections in the magnetic field. To do this, we expand the Landau function f_2 in equation (8), keeping linear terms in the vector potential \mathbf{A} ,

$$f_{\mathbf{p}+e\mathbf{A}, \mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}') = f_2(\mathbf{r}-\mathbf{r}') \frac{(\mathbf{p} \cdot \mathbf{p}')^2}{p_F^4} + 2e \frac{f_2(\mathbf{r}-\mathbf{r}')}{p_F^4} (\mathbf{p} \cdot \mathbf{p}') [\mathbf{A}(\mathbf{r}) \cdot (\mathbf{p} + \mathbf{p}')] . \quad (20)$$

With this expression, equation (8) reduces to

$$\delta \varepsilon_{\mathbf{p}}(\mathbf{q}, t) = \delta \varepsilon_{\mathbf{p}}^0(\mathbf{q}, t) + \delta \varepsilon_{\mathbf{p}}^A(\mathbf{q}, t), \quad (21)$$

where the first term has no contribution from the magnetic field, and is given by

$$\delta \varepsilon_{\mathbf{p}}^0(\mathbf{q}, t) = -i|f_2| \sum_{\mathbf{p}'} \left(\frac{\partial n_{\mathbf{p}'}^0}{\partial \varepsilon_{\mathbf{p}'}} \right) \left(\frac{\mathbf{p} \cdot \mathbf{p}'}{p_F^2} \right)^2 \nu_{\mathbf{p}'}(\mathbf{q}, t), \quad (22)$$

while the second term is linear in ω_c ,

$$\delta \varepsilon_{\mathbf{p}}^A(\mathbf{q}, t) = -2i \left(\frac{\omega_c}{v_F p_F} \right) \frac{(\kappa p_F)^2 F_2^2}{p_F^4 N(0)} \times \sum_{\mathbf{p}'} \left(\frac{\partial n_{\mathbf{p}'}^0}{\partial \varepsilon_{\mathbf{p}'}} \right) (\mathbf{p} \cdot \mathbf{p}') [(\mathbf{p} + \mathbf{p}') \times \mathbf{q}] \nu_{\mathbf{p}'}(\mathbf{q}, t), \quad (23)$$

where we have chosen the symmetric gauge $\mathbf{A} = (1/2)\mathbf{r} \times \mathbf{B}$. This term depends on the interaction range $(\kappa p_F)^2$,

then, for ultra-local interactions ($\kappa = 0$), it makes no contribution. On the other hand, the vectorial structure of equation (23) filters only contributions to the modes $\nu_{\pm 1}, \nu_{\pm 2}$. Therefore, Fourier transforming in \mathbf{p} we find for these modes,

$$\frac{\partial \nu_1}{\partial t} + i \left(\frac{v_F q}{2} \right) \left\{ \left[1 - 2(1-\alpha)^2 \left(\frac{\omega_c}{v_F q} \right)^2 (\kappa q)^2 \right] \nu_0 + \left[\alpha - 2(1-\alpha)^2 \left(\frac{\omega_c}{v_F q} \right)^2 (\kappa q)^2 \right] \nu_2 \right\} + i\omega_c \nu_1 = 0 \quad (24)$$

and

$$\frac{\partial \nu_2}{\partial t} + i \left(\frac{v_F q}{2} \right) \left\{ 1 - 4(1-\alpha)^2 \left(\frac{\omega_c}{v_F q} \right)^2 (\kappa q)^2 \right\} \nu_1 + i \left(\frac{v_F q}{2} \right) \nu_3 + 2i\alpha \omega_c \nu_2 = 0. \quad (25)$$

The equations for the modes ν_{-1} and ν_{-2} are easily obtained from equations (24) and (25) by changing $\ell \rightarrow -\ell$, and $\omega_c \rightarrow -\omega_c$. The dynamical equations for the rest of the modes are simply given by equation (18). Then, building symmetric and antisymmetric mode combinations, and deriving the evolution equations to get a second order system, we get for ν_2^+ ,

$$\frac{\partial^2 \nu_2^+}{\partial t^2} + \Omega^2 \nu_2^+ + \left(\frac{v_F q}{2} \right)^2 (\nu_0 + \nu_4^+) + \left(\frac{v_F q}{2} \right) \omega_c (2\nu_1^- + 3\nu_3^-) = 0 \quad (26)$$

with

$$\Omega^2 = 2\alpha \left(\frac{v_F q}{2} \right)^2 \left[1 + 8\alpha \left(\frac{\omega_c}{v_F q} \right)^2 \right] + \left(\frac{\kappa q}{2} \right)^2 (1-\alpha)^2 \omega_c^2. \quad (27)$$

As we have anticipated, the magnetic field mixes symmetric and antisymmetric modes. The first contribution to the frequency in equation (27) comes from the linear response theory, while the last term, proportional to the interaction range $\kappa q \ll 1$, is the first “correction” coming from equation (23).

Near the transition line $\Omega \rightarrow 0$, ν_0 and ν_4^+ are very rapid and stable modes, while the coupling with the antisymmetric modes are very weak. Thus, they do not modify the transition qualitatively. In the next section we will study these couplings in more detail. Therefore, ν_2^+ is unstable when $\Omega = 0$, leading to the transition line,

$$\left(\frac{\omega_c}{v_F q} \right)^2 = -\frac{1}{8} \frac{\alpha}{\alpha^2 + (1-\alpha)^2 \left(\frac{\kappa p_F}{4} \right)^2} \quad (28)$$

that we depict in figure (1). As expected, a magnetic field strongly reduces the phase space for Pomeranchuk instabilities. For small values of the magnetic field, the quantum critical point is shifted to greater attractive values of the interactions $\alpha < 0$, or $F_2 < -1$. Indeed, we observe a maximum value of the magnetic field

$$\left(\frac{\omega_c}{v_F q} \right)_{max}^2 \sim \frac{2}{(\kappa p_F)^2} \ll 1 \quad (29)$$

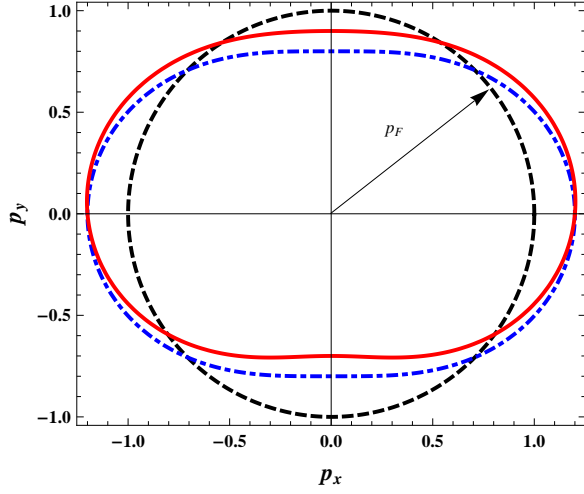


FIG. 2. Snapshot of the Fermi surface deformation, near the quantum critical point $\alpha = 0, \omega_c = 0$. The dash line is the actual Fermi surface, where we have normalized $p_F = 1$. The dash-dotted line is the elliptic deformation without magnetic field $\omega_c = 0$. The continuous line is the deformation in the presence of a small magnetic field. The vector \mathbf{q} is aligned with the p_x -axis, then the parity symmetry breaking in the p_y -axis is evident.

reached at $\alpha_{max} \sim -1 + 8/(\kappa p_F)^2$, above which, no Pomeranchuk instability is possible. Moreover, we observe a reentrant behavior of the disordered isotropic

phase for greater values of the attractive interaction.

IV. COLLECTIVE MODES NEAR THE QUANTUM CRITICAL POINT

We would like to analyze the behavior of the stable oscillation modes of the Fermi surface near the quantum critical point ($\alpha = 0, \omega_c = 0$). We are interest in the regime, $\alpha \ll 1$, and $\omega_c \ll v_F q \ll v_F p_F$.

We will focus in the unstable model ν_2^+ . This mode is directly coupled with ν_0, ν_4^+, ν_1^- and ν_3^- , through equation (26). The symmetric modes ν_0 and ν_4^+ are stable modes and oscillate very rapidly near the quantum critical point. Therefore, if we are interest in time scales larger than $(v_F q)^{-1}$, we can averaged them to zero. The antisymmetric modes ν_1^- and ν_3^- couple with ν_2^+ through a magnetic field ω_c , as a manifestation of parity symmetry breaking. Then, dismissing the symmetric couplings, ν_0, ν_4^+ , the remaining system $(\nu_2^+, \nu_1^-, \nu_3^-)$ is a close one. Defining the vector $\nu = (\nu_2^+, \nu_1^-, \nu_3^-)$, the collective modes satisfy,

$$\frac{\partial^2 \nu(q, t)}{\partial t^2} + M \nu(q, t) = 0, \quad (30)$$

where the matrix M takes the following form near the quantum critical point:

$$M = \begin{pmatrix} 2 \left(\frac{v_F q}{2}\right)^2 \alpha + \omega_c^2 \left(\frac{\kappa q}{2}\right)^2 & \left(\frac{v_F q}{2}\right) \omega_c & 3 \left(\frac{v_F q}{2}\right) \omega_c \\ \left(\frac{2\omega_c}{v_F q}\right) \left[\left(\frac{v_F q}{2}\right)^2 \alpha + \left(\frac{\omega_c \kappa q}{2}\right)^2\right] & \left(\frac{v_F q}{2}\right)^2 \alpha + \omega_c^2 \left(1 + \left(\frac{\kappa q}{2}\right)^2\right) & \left(\frac{v_F q}{2}\right)^2 \alpha + \left(\frac{\omega_c \kappa q}{2}\right)^2 \\ 5\alpha \omega_c \left(\frac{v_F q}{2}\right) & \alpha \left(\frac{v_F q}{2}\right)^2 & \left(\frac{v_F q}{2}\right)^2 \end{pmatrix}. \quad (31)$$

It is instructive to analyze two different paths when approaching the quantum critical point. In the case of zero applied magnetic field ($\omega_c = 0, \alpha \rightarrow 0$), the antisymmetric modes completely decouple from the symmetric ones, due to parity symmetry. Then, the ν_2^+ frequency coincides with equation (17). However, when approaching the quantum critical point lowering the magnetic field ($\alpha = 0, \omega_c \rightarrow 0$), the matrix $M_c = \lim_{\alpha \rightarrow 0} M$ takes the form

$$M_c = \begin{pmatrix} \omega_c^2 \left(\frac{\kappa q}{2}\right)^2 & \left(\frac{v_F q}{2}\right) \omega_c & 3 \left(\frac{v_F q}{2}\right) \omega_c \\ \left(\frac{2\omega_c}{v_F q}\right) \left(\frac{\omega_c \kappa q}{2}\right)^2 & \omega_c^2 \left(1 + \left(\frac{\kappa q}{2}\right)^2\right) & \left(\frac{\omega_c \kappa q}{2}\right)^2 \\ 0 & 0 & \left(\frac{v_F q}{2}\right)^2 \end{pmatrix}. \quad (32)$$

In order to find the normal modes, we diagonalize M_c

obtaining the eigen-values

$$\lambda_1 = \left(\frac{\kappa q}{2}\right)^4 \omega_c^2, \quad \lambda_2 = \omega_c^2, \quad \lambda_3 = \left(\frac{v_F q}{2}\right)^2, \quad (33)$$

with the corresponding eigen-vector matrix

$$A = \begin{pmatrix} 1 & 2 \left(\frac{\omega_c}{v_F q}\right) \left(\frac{\kappa q}{2}\right)^2 & 0 \\ -2 \left(\frac{\omega_c}{v_F q}\right) \left(\frac{\kappa q}{2}\right)^2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (34)$$

Thus, it is clear from equations (34) that the mode ν_3^- decouples for $\alpha = 0$ and it is a rapid mode, oscillating with frequency $v_F q/2$. On the other hand, the modes ν_2^+ and ν_1^- are slow modes, coupled by the small quantities $(\omega_c/v_F q) \ll 1$ and $(\kappa q/2)^2 \ll 1$. The former is related

with the cyclotron frequency that it should be smaller than the frequency of a typical perturbation $(v_F q)^{-1}$, while the latter is related with the interaction range that should be much smaller than the typical length of the Fermi surface perturbation q^{-1} .

Therefore, very near the Pomeranchuk instability ($\alpha = 0, \omega_c \ll v_F q$), the Fermi surface fluctuates following the equation

$$\begin{aligned} \delta k_F = & \nu_2^i \left\{ \cos(2\theta) + 2 \left(\frac{\omega_c}{v_F q} \right) \left(\frac{\kappa q}{2} \right)^2 \sin \theta \right\} \times \\ & \times \cos \left[\left(\frac{\kappa q}{2} \right)^2 \omega_c t + \varphi_1 \right] + \\ & + \nu_{-1}^i \left\{ \sin(\theta) - 2 \left(\frac{\omega_c}{v_F q} \right) \left(\frac{\kappa q}{2} \right)^2 \cos(2\theta) \right\} \times \\ & \times \cos [\omega_c t + \varphi_2] + \\ & + \nu_{-3}^i \sin(3\theta) \cos \left[\left(\frac{v_F q}{2} \right) t + \varphi_3 \right], \end{aligned} \quad (35)$$

where $\nu_2^i, \nu_{-1}^i, \nu_{-3}^i$ and $\varphi_1, \varphi_2, \varphi_3$ are the initial amplitudes and phases, respectively.

We see that there are two slow modes that oscillates with frequencies proportional to ω_c . The slowest mode (λ_1 in equation (33)) is related with ν_2^+ , and it is responsible for the Pomeranchuk instability when $\omega_c \rightarrow 0$. On the other hand, the mode associated with the eigen-value λ_2 is related with the anti-symmetric mode ν_1^- . However, this mode is not unstable at the quantum critical point since, when $\omega_c \rightarrow 0$, not only its frequency goes to zero, but also its velocity $\partial \nu_1^- / \partial t \rightarrow 0$, implying a constant mode at the quantum critical point, decoupled from any other symmetric mode.

In figure (2) we show a snapshot of the Fermi surface near the Pomeranchuk instability, where we have chosen the initial conditions $\nu_2^i = 0.2$ and $\nu_{-1}^i = \nu_{-3}^i = 0$. The circular dash line is the actual isotropic Fermi surface. The dash-dotted line shows a ellipse, which indicates the usual deformation with nematic symmetry in the absence of magnetic field, while the continuous line is the deformation of the Fermi surface in the presence of a magnetic field. As expected, we observe a parity breaking in the axis $\mathbf{q} \times \mathbf{B}$ (in this case in p_y , since we have chosen \mathbf{q} pointing in the p_x direction). This is a tiny effect proportional to $(\omega_c / v_F q) (\kappa q / 2)^2 \ll 1$. In the figure we have artificially amplified this parameter, in order to make the effect of the magnetic field visible.

We have re-done all the calculations of this section considering also the couplings with the symmetric modes ν_0 and ν_4^+ . In this case, it is not possible to analytically solve the resulting 9×9 linear system. However, making a numerical analysis, we did not found any relevant deviation from the simplified calculation shown. This confirms in some way that the stable rapid modes do not participate in the instability process, very near the quantum critical point.

V. CONCLUSIONS

We have analyzed the behavior of a two-dimensional Fermi liquid submitted to an external magnetic field, near a Pomeranchuk instability triggered by the Landau parameter F_2 in the charged sector. We have considered a simple model in which the only interaction is given by the Landau parameter F_2 . The presence of other interactions does not modify the results qualitatively provided they are all stable, *i. e.*, distant from any other Pomeranchuk instability.

We have studied the Fermi surface stability, approaching the critical region from the isotropic phase, where the Landau theory of Fermi liquids can be used safely. We have studied collective modes using the semi-classical Landau-Silin equation. Usually, this equation was studied in the linear response approximation to analyze plasma modes in charged Fermi liquids. However, near a Pomeranchuk instability this approximation is not sufficient. The reason is that the quantum critical point is controlled by two parameters, $\alpha = 1 + F_2$ and ω_c . The leading order correction in the magnetic field is proportional to $\alpha \omega_c$. Thus, near the quantum critical point ($\alpha = 0, \omega_c = 0$), corrections proportional to α^2 and ω_c^2 are of the same order and cannot be neglected. Therefore, we need to go to quadratic order in the magnetic field to consistently treat the neighborhood of the quantum critical point.

There are essentially two scales in the theory. A short distance scale given by the inverse of the Fermi momentum p_F^{-1} and a long distance scale $q^{-1} \gg p_F^{-1}$ given by the momentum of the local Fermi surface perturbations. Our approach makes sense provided the interaction range is greater than the inter-particle distance p_F^{-1} but shorter than the long distance perturbation q^{-1} .

Identifying the slowest collective mode, it is possible to compute the transition line given in figure (1). We observe an upper limit value for the magnetic field $\omega_c \sim v_F q / \kappa p_F$ over which the Pomeranchuk instability is completely suppressed. For smaller values of the magnetic field, we observe that the instability is shift to stronger values of the attractive interaction. Moreover, it is observed a reentrant behavior of the isotropic phase for even stronger attractive interactions. Reentrant behavior has posed challenges to microscopic theoretical physics in a variety of condensed matter systems¹⁴⁻²¹. This phenomenon is characterized by the reappearance of a less ordered phase, following a more ordered one, as a control parameter (for example, temperature, pressure, chemical doping, magnetic field) is varied. It appears that the reentrance phenomenon also occurs, as we report in this paper, in the phase diagram for the Pomeranchuk instability $\ell = 2$ in the presence of a small magnetic field, at least as indicated from our semi-classical approach calculations. Essentially, the reentrant phenomenon can be produced by the increasing of entropy due to disorder or due to the presence of additional degrees of freedom. We will leave the study of these phenomena for a further

work.

We have also studied collective modes couplings near the critical region. We have identified the ν_2^+ mode as the mainly unstable mode when the quantum critical point is approached. That means that the main contribution to the Fermi surface deformation has elliptic (nematic) symmetry. However, the magnetic field couples this mode with the antisymmetric ones ν_1^- and ν_3^- . The anti-symmetric ν_3^- is a rapid mode oscillating with frequency $v_F q/2$ and it does not participate of the instability process. On the other hand, ν_1^- is a slow mode, however quicker than ν_2^+ , since it oscillates with the cyclotron frequency ω_c . Even though its frequency goes to zero at the quantum critical point, it does not represent a real Pomeranchuk instability since, on one hand, its coupling with ν_2^+ also goes to zero with the magnetic field and, on the other hand, not only its frequency but also its velocity goes to zero as $\omega_c \rightarrow 0$. However, it has an important effect on the Fermi surface deformation of the unstable mode since its coupling is a direct consequence of parity breaking, producing a contribution that breaks nematic symmetry as shown in figure (2). In fact, near the quantum critical point the slowest mode is invariant

under the combined transformation $\theta \rightarrow \theta + \pi, \omega_c \rightarrow -\omega_c$.

In order to have a complete picture of the isotropic-nematic phase transition under the influence of a magnetic field, it is necessary to study the ordered phase. However, the Landau theory of Fermi liquids is not the best approach to do that, since in the absence of a magnetic field, the ordered nematic phase is also critical⁴ and the quasi-particle picture is no more adequate to treat this problem. Conversely, it is necessary to face it with other approaches like non-perturbative calculations on specific fermionic models.

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